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Qu. 30. *Answered by Mr. O'Shannessy, Albany.*

Let s be the sum, and p the product of the req. numbers; then $s^2 - 2p = 41$, and $s^3 - 3sp = 189$; substitute in this latter equ. the value of p found in the former, and we have $s^3 - 123s + 378 = 0$, from whence $s = 4$ and $p = 20$; therefore, 4 and 5 are the req numbers.

Qu. 31. *Answered by Y., New-Haven.*

Let x and y be the req. num. By the 3d condition, $x^2 + y = y^2 + x$, or $x^2 - y^2 = x - y$; by division, $x + y = 1$, which being a perfect cube, ans. the first condition. We have now only to make $x.(1 - x)$ a sqr. assume it $= y^2$, and x is found $= \frac{1}{2} + \surd(\frac{1}{4} - y^2)$. To make this surd rational, let $\frac{1}{4} - y^2 = (\frac{1}{2} - uy)^2$ and y is found $= u \div (u^2 + 1)$. This, by substitution, gives $x = \dfrac{1}{u^2 + 1}$, & $y = \dfrac{u^2}{u^2 + 1}$, in which u may be of any value whatever, whole or broken, positive or negative.

Qu. 32. *Answered by Mr. C. Davis.*

Let x denote the degrees between the minute hand and 12 o'clock; $x + 5$ will be the distance of the hour hand from the same point. But the hour hand comes to 10, when the minute hand arrives at 12, leaving a diff. of 60°, and moves with $\frac{1}{12}$ the velocity, theref. $\frac{1}{12} x = x + 5 - 60$, and $x = \frac{12}{11}$, and the time, allowing 1' to 6°, is either 10 or 11 $\frac{9}{11}$ min. before 10.

Qu. 33. *Answered by Y.*

Because 5,600 dollars are lent at first, and 400 reserved, the time req. will be the same as if 14 dollars be supposed lent, and 1 reserved. The diff. between reserving 1 dollar at the end of each year, and allowing the whole to go on at com. interest, will evidently be 1 dollar for one year, $1 + 1{,}06$ for 2 years, $1 + 1{,}06 + (1.06)^2$ for 3 years, &c. Put x for the number of years req. and the sum of this series to $x - 1$ terms, or, $\frac{(1.06)^x - 1}{.06}$ is equal to the amount of 14 dol. for x years $= 14.\ (1.06)^x$; this equ. reduced, gives $x = \frac{\log 6\frac{1}{4}}{\log 1{,}06} =$ 31.450 years.

Qu. 34. *Answered by Y.*

The figure formed by the intersection of two equal cylinders in the manner described in the qu. is evidently the common groin, the solidity of which, when the diam. is denoted by a, is $\frac{2}{3}\,a^3$, and its surface $= 4\,a^2$.

Qu. 35. *Answered by Zero.*

Put $L =$ length of the second's pendulum in lat. of London; and let gravity be increased by its nth part. Now the lengths of pendulums are as the force of gravity, when the times of vibration are the same; theref. $1 : \frac{n+1}{n} :: L : \frac{n+1}{n}\,.\,L$ $=$ length of the pend. req.; and when $n = 100$, the length required is 39.592 inches. Again, gravity being the same, the lengths of pend. are as the sqrs. of the times of vibration; hence $9 : 25 :: 39.592 : 109.97 =$ length of the pendulum vibrating 3 times in 5 seconds.

Qu. 36. *Answered by Y.*

Let x be the secant of the req. arc. and r the rad.; its cosine will be $r^2 \div x$ and by *Eu.* VI. 3 . $r + x : r :: \sqrt{(x^2 - r^2)} : \frac{r^2}{x}$; whence $x^4 - 2\,r^2\,x^2 - 2\,r^3\,x = r^4$, an equa. which, like all other cubics that cannot be depressed by resolving into factors, can only be constructed by means of a parabola, or some other conic section.

Qu. 37. *Answered by Y.*

Put the given solidity of the paraboloid $= \frac{1}{2} p b$, ($p =$ 3.14, &c.) $z =$ the parameter, and the abscissa $= x$; the ordinate will be $= \sqrt{\frac{b}{x}}$. The rad. of a sphere inscribed in a paraboloid is the same with that of a circle inscribed in the generating parabola, and this is readily found by the nature of the figure drawn for the purpose, to be $= \sqrt{xz} - \frac{1}{2} z$, which by substitution becomes $\sqrt{\frac{b}{x} - \frac{b}{2x^2}}$. Hence we have to make $\frac{4}{3} p \left(\sqrt{\frac{b}{x} - \frac{b}{2x^2}}\right)^3$, or $\sqrt{\frac{b}{x} - \frac{b}{2x^2}} =$ a max. Taking the differential $= 0$, and reducing, x is found $= 4z$.

Qu. 38. *Answered by Zero.*

The horizontal axis coincides with the line made up of all the centres of pressure of the vertical lines into which the gate may be conceived to be divided. Now (by Gregory's Mech. vol. 1. pa. 359. or pa. 389. Marrat's Mech.) the dist. of this line from the upper edge of the gate is the same as that of the centre of percussion. Hence the axis must be $= \frac{2}{3}$ of the dist. to the bottom, or the whole length of the gate $= 6$ feet. [This prob. will suggest to the practical engineer an improved method of hanging *flood gates*, so that water may be let off with much greater facility than when they are hung according to the common method.—Ed.]

Qu. 39. *Answered.*

By a little consideration the price of each is $= 6$ pence, and the whole cost 5 shillings.

Qu. 40. *Answered.*

Let $x =$ length and $y =$ breadth; then the first condition gives $2x + 3y = 53$, and the second, $3x + 2y = 62$; hence $x = 14$, and $y = 10$.

Qu. 41. *Answered.*

Let x, y and z denote the successive numbers req.; by the qu. $x + \frac{1}{2} y + \frac{1}{2} z = y + \frac{1}{3} x + \frac{1}{3} z = z + \frac{1}{4} x + \frac{1}{4} y = 901$; from whence $x = 265$, $y = 583$, and $z = 689$.

Qu. 42. *Answered by Zero.*

Put the semitransverse axis of the ellipse $= a$, and the dist. of its centre from the bottom of the cone $= x$; then, the semiconjugate axis is found $= \frac{1}{2} \sqrt{(x^2 - a^2)}$, and the alt. of the seg. immersed is $= 3a - x$; hence the solidity $= \frac{x . (x^2 - a^2) \cdot (3a - x)^2}{12 p a^2}$, is to be a max. Making its differential $= 0$, we get $5 x^4 - 24 a x^3 + 24 a^2 x^2 + 12 a^3 x - 9a^4 = 0$; or (putting $x = ay$) $5 y^4 - 24 y^3 + 24 y^2 + 12 y - 9 = 0$; from this equ. y is found $= 1.949836$, and $x = 5.849658$, whence the solidity of the spheroid in inches $= 79.22634$.

Qu. 43. *Answered by Y.*

Let $x^2 - y$ be assumed $= (x - 1)^2$ and $x^2 - z = (x - 2)^2$ y is obtained $= 2x - 1$, and $z = 4 (x - 1)$. Since $y^2 - z$ is to be a sqr. we have to make by substitution $4 x^2 - 8 x + 5$ a sqr.; assume its root $= 2x - a$, which gives $x = \frac{a^2 - 5}{4a - 8}$. Again, since $y^2 - x$ is to be a sqr. by substitution and reduction, $a^4 - 5a^3 + 4a^2 + 9a - 9 =$ a square. Let $a^2 + pa + q$ be its root; take its square, and assume the co-efficients of the second and third terms on each side equal, p is found $= - \frac{5}{2}$, and $q = - \frac{9}{8}$; from which $9a - 9 = \frac{45 a}{8} + \frac{81}{64}$, and $a = \frac{73}{24}$: hence by substitution $x = \frac{2449}{2400}$, $y = \frac{1249}{1200}$, and $z = \frac{49}{600}$, which numbers have the req. conditions.

Qu. 44. *Answered by Y.*

Consider a vertical line bisecting the curve as the axis of the abscissas, and the original position of the rod as that of the ordinates. Put the dist. from any point in the curve to the axis of the abscissas $= y$, and the part of this axis intercepted between y and the axis of the ordinates $= x$, and $p = 3.14159$, denote the length of the rod by unity, and the equa. of the curve is found to be $\frac{1}{p}$ arc. (sin. $= x) + \sqrt{(1 - x^2)} - \frac{1}{2} = y$.

Hence the differential of the area $= \frac{1}{p}$ arc $(\sin = x) \times dx + dx \sqrt{(1 - x^2)} - \frac{1}{2} dx$. To find the integral of the first part, put arc $(\sin = x) = z$; then $\int z dx = zx - \int x dz$. But if $v = \sqrt{(1 - x^2)}$, $x : 1 :: - dv : dz$, so that $x dz = - dv$, and $\int x dz = - \sqrt{(1 - x^2)}$, theref. the integral of the first term is $\frac{1}{p} \sqrt{(1 - x^2)} + \frac{x}{p}$ arc $(\sin = x)$. That of the 2d term is $\frac{1}{2}$ arc $(\sin = x) + \frac{1}{2} x \sqrt{(1 - x^2)}$; and that of the 3d is $= - \frac{1}{2} x$. The correction reqd. in the sum of these fluents by the supposition of $x = 0$, when the area begins, is $= - \frac{1}{p}$. Let 1 be substituted for x, which is its greatest value, and the integral for half the area of the curve, comes out $= \frac{p}{4} - \frac{1}{p}$. If r = length of the rod, the whole area described by it will be $r^2 \times \left(\frac{p}{2} - \frac{2}{p}\right)$. This curve is a portion of a curtate cycloid.

Qu. 45. *Answered by Mr. Laidlaw, Brooklyn.*

Denote the given num. by a, b, and $b^2 - c$, and put $y^{\frac{3}{4}} = m$; then, the 3 equa. are easily reduced to $m^4 - 2bm^2 + \sqrt{(a - m)} + c = 0$. Restoring the numbers we find $m = 12.8186$ nearly, and then $x = 40$, $y = 30$, and $z = 20$.

Qu. 46. *Answered by Y.*

Let the required digits be denoted by x, $x - y$, and $x - 2y$; then from the first condition x is found $= 4y$. and from the second $y = 1$; hence the reqd. number is 432.

Qu. 47. *Answered by Mr. J. Laidlaw, Brooklyn.*

Let x = num. of hens, then $\frac{2}{3} x$ = num. of cheeses, and $\frac{1}{3} x^2$ = the whole num. of eggs, theref. $9 : \frac{1}{3} x :: \frac{1}{3} x^2 : 72$; whence x is found $= 72$.

Qu. 48. *Answered by C. West Point.*

Let us conceive an upright cone, and 3 parallel lines intersecting its base, *viz.* a diam. a secant, and a tangent; through

these 3 lines and the vertex of the cone, draw 3 planes, the 1st of which will be a plane diam. the 2d a secant, and the 3d a tangent plane; they will evidently intersect each other in a common line parallel to the 3 first lines, and passing through the vertex. Let us now intersect the cone by any plane perp. to the plane diam.; it cuts the 3 planes in 3 lines which meet at the intersection of the cutting plane, and the horizontal line of the vertex. Of these 3 one will be the greater axe, another a secant, and the 3d a tan. to the conic section. Draw now the radius perp. to the tan. to the circle, and two other radii to the intersection of the secant, and conceive 3 planes through these 3 lines and the vertex; they will cut the plane of the conic section in 3 lines, one of which being the intersection of 2 planes perp. to the plane diam. will be an ordinate to the *curve*, and the two others will make equal angles with it, and, consequently, with the axis; for the two radii of the circle, making equal angles with the first diam., the two corresponding lines in the plane of the curve are the intersections of two planes making equal angles with the plane diam. by a plane perp. to it.

To the Editor of the Scientific Journal.

Sir,

As the writer in your last No. who subscribes himself C., has denied the possibility of the curve to which question 23 relates, and has thus implied that the solutions given by your other correspondents are inaccurate, it seems no more than fair that they should have the opportunity of vindicating themselves. The most superficial examination of his solution will render it evident that instead of *proving* the line which bears a constant ratio to its subtangent to be of the first order, he takes it for granted. He first assumes dy constant, and afterwards, in taking the differentials of the equation $\frac{dx}{dz} = p$, assumes dz constant, otherwise the differential equation would be $\frac{d^2x}{dz} - \frac{d^2zdx}{dz^2} = dp$, instead of $\frac{d^2x}{dz} = dp$. By assuming dy and dz constant, he has indeed shown that the required property belongs to a straight line, (although in the single case where $m = 1$, the solution fails, for a parallel to the axis can have no

subtangent,) but he has by no means proved that it does not also belong to a curve. That it does so in fact, may be rendered evident. C. will not deny that if a perpendicular be drawn from a given straight line, a series of equal straight lines may be drawn from the extremity of this perp., such that, if any one of them be produced, and also a perp. be let fall to meet the given line, the intercepted portion of it will be equal to the corresponding part of the polygon. Indeed, the direction in which each side of the polygon must be drawn, can be readily calculated. Now as the lines composing it can be taken of any magnitude, however small, it evidently follows that there is a continued curve, possessing the same property.

To prevent any misapprehension which may arise from comparing the solutions of this problem in your last No. with that in the preceding, it may not be improper to state that it was not till after obtaining an equation between y and the differential co-efficient p, equivalent to those of B. and Analyticus, that I hazarded the opinion that the relation between the curve and ordinate could not be expressed in "finite algebraic terms." Finite expressions, even when transcendental, are doubtless to be generally preferred to series; but in somes cases the finite expression may be so complex, and the series so simple, that room may be left for a difference of opinion as to their comparative elegance.

Y.

[*The solutions to the following questions must arrive before the first of September next. No. IX. will be published on the 1st of October.*]

Qu. 49. *By Zero.*

Show how to divide any cube number, as n^3, into 3 cubes.

Qu. 50. *By Galilei Galileo.*

Given the base CB the alt. BA, and length AC, of an inclined plane; to find in AC, a part $=$ AB, through which a body would descend in the same time that another body would fall freely from rest at A through the alt. AB?

Qu. 51. *By Y.*

Rectify the curve whose equation is $y = \sin x$, by means of an elliptic arc; and find the superficies generated by a revolution of the curve round its axis, in finite terms?

Qu. 52. *By Mr. W. Marrat, New-York.*

Some writers on Navigation assert that the principles on which *plane sailing* is founded are erroneous, the earth being considered as a globe; is this assertion true or false?

Qu 53. *By Mr. J. Campbell, Teacher, New-York.*

Given, $x^x = a$, to find the value of x?

Qu. 54. *By Mr. W. Marrat.*

A cylindrical vessel, full of water, and closed at the top, will just stand upon an inclined plane without falling over; if now a very small hole be made in the side, at the lowest point of the *upper* end of the vessel, to what dist. from the foot of the cylinder will the water spout upon the plane; the length of the vessel, and diameter of its base being 40 and 30 inches respectively?

The *Mathematical* part of the "Scientific Journal" will in future be published *quarterly*, by a Committee of the Friends of Science, who are of opinion that its contents will be useful, at least to themselves. If it meet with a sufficient number of purchasers, it will be enlarged at the same price. Measures, however, have been adopted to insure its continuance. Diagrams will be given when necessary. Any gentleman who wishes to have the work sent to him, must send in advance to the Editor *one dollar* for a year's subscription. Communications of every kind (post paid) must be directed to the "Editor of the Scientific Journal, 20 John-street, New-York." New questions and solutions will meet with proper attention. Those Gentlemen to whom the Editor is indebted for subscriptions paid to him in advance, will be considered as subscribers till the subscription is run out, and the numbers will be sent as usual. Those who have not paid *any subscription*, or are in arrears, will gratify him by an immediate remittance. Nearly all the above questions were answered by all the gentlemen whose names are in the list at the end of No. VI.

Mr. F. Nichols has ready for the press the third edition of Playfair's Geometry, or Euclid revised and improved; also an abridgement of Playfair's Geometry, for the use of Schools.